Civil and Environmental Systems Engineering (2nd Edition)

Chapter 5, Problem 9E ☐ Bookmark Show all steps: ON

Problem

Consider the following multiple-objective linear program:

Maximize Z1 = 9x1 + 6x2

Maximize Z2 = 3x1 + 2x2

Subject to: 3x1 + 2x2 ≤ 30

−6x1 + 3x2 ≤ 12

4x1 + 2x2 ≤ 24

x2 ≤ 6

x1, x2 ≥ 0.

(a) Plot the feasible region in decision space for this problem.

(b) Plot the corresponding feasible region in objective space for this problem. For each extreme point indicate if it is a noninferior or a dominated solution.

(c) Use the constraint method (graphically) to generate an approximation of the noninferior set having 6 noninferior solutions evenly spaced along the Z1 axis.

(d) Use the constraint method (graphically) to generate an approximation of the noninferior set having 6 noninferior solutions evenly spaced along the Z2 axis.

Step-by-step solution

Step 1 of 25

Show the multiple objective linear programs:

$$\text{Maximize } Z_1 = 9x_1 + 6x_2$$
$$\text{Minimize } Z_2 = 3x_1 - 2x_2$$

Consider the following constraints:

$$3x_1 + 2x_2 \le 30 \quad \ldots\ldots (1)$$
$$-6x_1 + 3x_2 \le 12 \quad \ldots\ldots (2)$$
$$4x_1 - 2x_2 \le 24 \quad \ldots\ldots (3)$$
$$x_2 \le 6 \quad \ldots\ldots (4)$$
$$x_1, x_2 \ge 0$$

Comment

Step 2 of 25

Introduce the slack variables in Equations (1), (2), (3), and (4):

$$3x_1 + 2x_2 + s_1 = 30 \quad \ldots\ldots (5)$$
$$-6x_1 + 3x_2 + s_2 = 12 \quad \ldots\ldots (6)$$
$$4x_1 - 2x_2 + s_3 = 24 \quad \ldots\ldots (7)$$
$$x_2 + s_4 = 6 \quad \ldots\ldots (8)$$

Write the objective equations:

$$\text{Maximize } Z_1 = 9x_1 + 6x_2 \quad \ldots\ldots (9)$$
$$\text{Minimize } Z_2 = 3x_1 - 2x_2 \quad \ldots\ldots (10)$$

Comment

Step 3 of 25

Construct the feasible region in objective space using the following calculation:

Consider the constraint:

$$3x_1 + 2x_2 \le 30$$

Assume the slack (s_1) as zero and rewrite the constraint:

$$3x_1 + 2x_2 = 30 \quad \ldots\ldots (11)$$

Tabulate the coordinates of x_1 and x_2 of Equation (11) as in Table (1):

x_1	0	2	4	6
x_2	15	12	9	6

Comment

Step 4 of 25

Consider the constraint:

$$-6x_1 + 3x_2 \le 12$$

Assume the slack (s_2) as zero and rewrite the constraint:

$$-6x_1 + 3x_2 = 12 \quad \ldots\ldots (12)$$

Tabulate the coordinates of x_1 and x_2 of Equation (12) as in Table (2):

x_1	0	2	4
x_2	4	8	12

Comment

Step 5 of 25

Consider the constraint:

$$4x_1 - 2x_2 \le 24$$

Assume the slack (s_3) as zero and rewrite the constraint:

$$4x_1 - 2x_2 = 24 \quad \ldots\ldots (13)$$

Tabulate the coordinates of x_1 and x_2 of Equation (13) as in Table (3):

x_1	6	8	10
x_2	0	4	8

Comment

Step 6 of 25

Consider the constraint:

$$x_2 \le 6$$

Assume the slack (s_4) as zero and rewrite the constraint:

$$x_2 = 6 \quad \ldots\ldots (14)$$

Plot the graph using x_1, x_2 coordinates from Tables (1), (2), and (3).

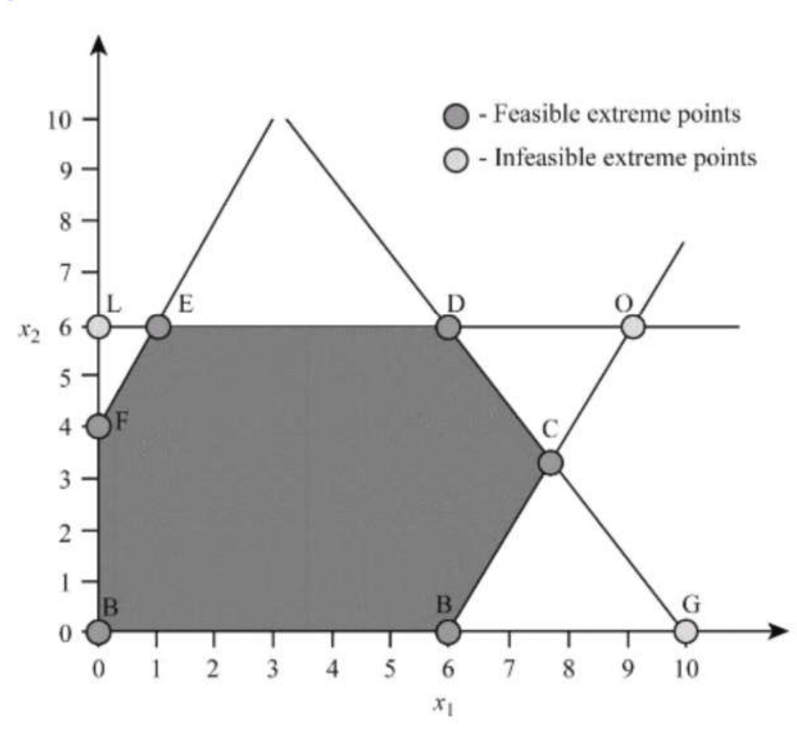
Four straight lines can be obtained from Equations (11), (12), (13), and (14).

Highlight the bounded area.

Comment

Step 7 of 25

Show the feasible region in decision space with solutions that optimize Z_1 and Z_2 as in Figure (1):



Comment

Step 8 of 25

From Figure (1):
A, B, C, D, E, and F are feasible extreme points.
L and O are infeasible extreme points.
The coordinates of x_1 and x_2 of points A, B, C, D, E, and F are obtained from Figure (1).

Find the slack variable 1 (s_1) at point A using Equation (5):

$$3x_1 + 2x_2 + s_1 = 30$$

Substitute 0 for x_1 and 0 for x_2:

$$3\times(0) + 2\times(0) + s_1 = 30$$
$$s_1 = 30$$

At point B:

$$3\times(6) + 2\times(0) + s_1 = 30$$
$$s_1 = 12$$

At point C:

$$3\times(7.71) + 2\times(3.43) + s_1 = 30$$
$$s_1 = 0$$

Comment

Step 9 of 25

At point D:

$$3\times(6) + 2\times(6) + s_1 = 30$$
$$s_1 = 0$$

At point E:

$$3\times(1) + 2\times(6) + s_1 = 30$$
$$s_1 = 15$$

At point F:

$$3\times(0) + 2\times(4) + s_1 = 60$$
$$s_1 = 52$$

Similarly, find the slack variable 2 (s_2), slack variable 3 (s_3), and slack variable 4 (s_4) for points A, B, C, D, E, and F using Equations (6), (7), and (8).

Comment

Step 10 of 25

Find the maximize value (Z_1) at point A using Equation (9):

$$\text{Maximize } Z_1 = 9x_1 + 6x_2$$

Substitute 0 for x_1 and 0 for x_2:

$$Z_1 = (9\times 0) + (6\times 0)$$
$$= 0$$

At point B:

$$Z_1 = (9\times 6) + (6\times 0)$$
$$= 54$$

At point C:

$$Z_1 = (9\times 7.71) + (6\times 3.43)$$
$$= 90$$

At point D:

$$Z_1 = (9\times 6) + (6\times 6)$$
$$= 90$$

Comment

Step 11 of 25

At point E:

$$Z_1 = (9\times 1) + (6\times 6)$$
$$= 45$$

At point F:

$$Z_1 = (9\times 0) + (6\times 4)$$
$$= 24$$

Comment

Step 12 of 25

Similarly, find the minimize value (Z_2) for points A, B, C, D, E, and F using Equation (10).

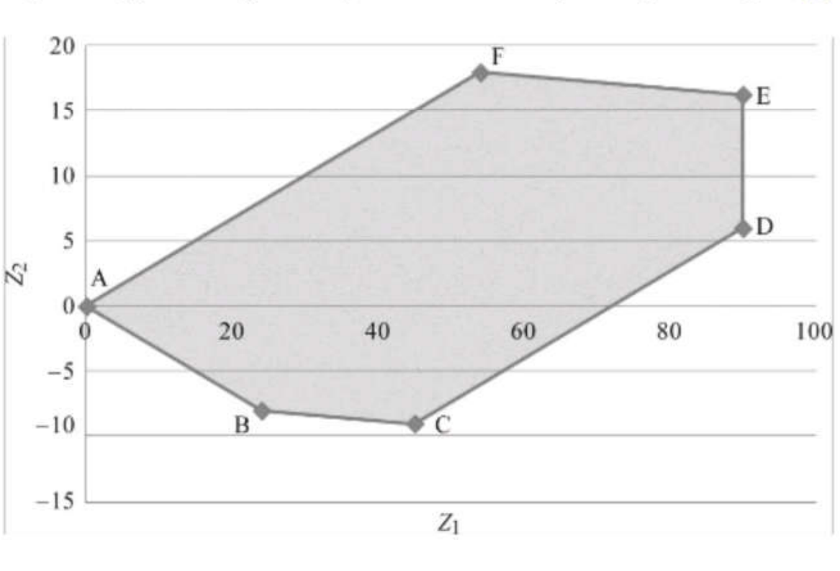
Tabulate the values of objectives Z_1 and Z_2 for all extreme points as in Table (4).

Point	x_1	x_2	s_1	s_2	s_3	s_4	z_1	z_2
A	0	0	30	12	24	6	0	0
B	6	0	12	48	0	6	54	18
C	7.71	3.43	0	48	0	2.57	90	16.29
D	6	6	0	30	12	0	90	6
E	1	6	15	0	32	0	45	−9
F	0	4	22	0	32	2	24	−8

Comment

Step 13 of 25

Show the feasible region in objective space by plotting all basic feasible solutions from decision space mapped through the objective functions Z_1 and Z_2 as in Figure (2).



Comment

Step 14 of 25

Approximation of the non-inferior set having non-inferior solutions along the Z_1 axis by using the constraint method:

Construct the payoff table as in Table (5):

Solution	x_1	x_2	$Z_1 = 9x_1 + 6x_2$	$Z_2 = 3x_1 - 2x_2$
x^1	6	6	90	6
x^2	1	6	45	−9

Comment

Step 15 of 25

From Table (5):
To optimize objective function Z_1, we need to move objective function Z_2 into the constraint set and solve the problem.

Hence,

$$L_{\max} = 6$$
$$L_{\min} = -9$$

Denote: $L_{\max}$ is largest value in the payoff table corresponding to the objective function Z_2 and $L_{\min}$ is minimum value in the payoff table corresponding to the objective function Z_2.

$$L_k = L_{\min} + \left[\frac{t}{r-1}\right](L_{\max} - L_{\min}) \quad \ldots\ldots (15)$$

Denote: L_k is true non-inferior generation variable, r is the number of non-inferior solutions, and t is the number of times.

Comment

Step 16 of 25

Substitute -9 for $L_{\min}$, 6 for $L_{\max}$, 6 for r, and 0 for t.

$$L_1 = -9 + \left[\frac{0}{6-1}\right]\times(6-(-9))$$
$$L_1 = -9$$

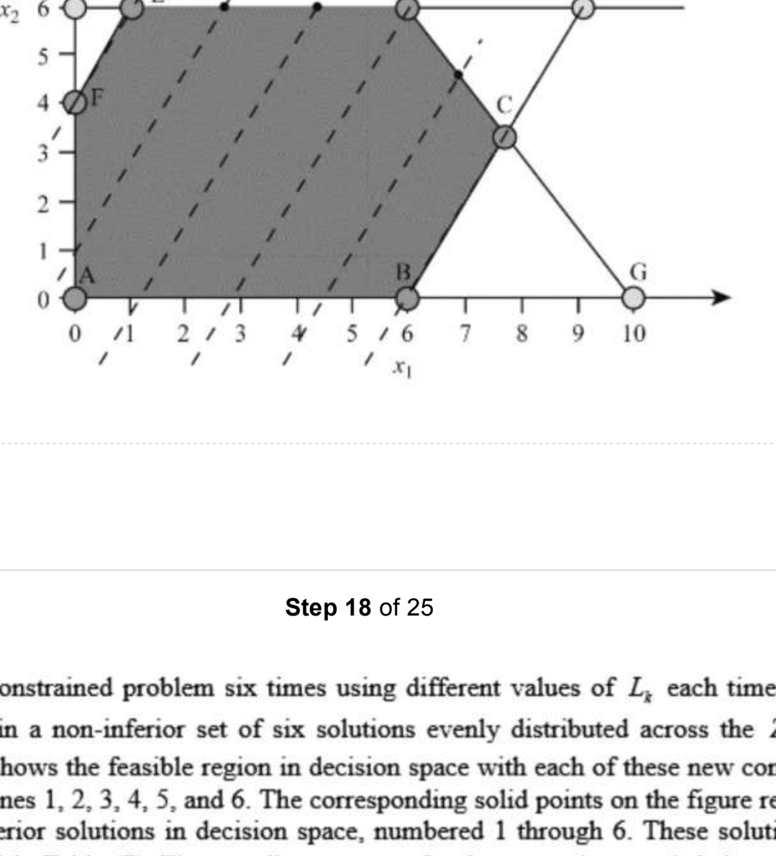
Similarly, find the L_k values for $t = 1, 2, 3, 4,$ and 5.

$$L_2 = -9 + \left[\frac{1}{6-1}\right]\times(6-(-9))$$
$$= -6$$
$$L_3 = -9 + \left[\frac{2}{6-1}\right]\times(6-(-9))$$
$$= -3$$
$$L_4 = -9 + \left[\frac{3}{6-1}\right]\times(6-(-9))$$
$$= 0$$
$$L_5 = -9 + \left[\frac{4}{6-1}\right]\times(6-(-9))$$
$$= 3$$
$$L_6 = -9 + \left[\frac{5}{6-1}\right]\times(6-(-9))$$
$$= 6$$

Comment

Step 17 of 25

Show the feasible region in decision space with Z_2 constrained to six different values of L_k as shown in Figure (3).



Comment

Step 18 of 25

Solve the constrained problem six times using different values of L_k each time, which will result in a non-inferior set of six solutions evenly distributed across the Z_1 axis. Figure (3) shows the feasible region in decision space with each of these new constraints plotted as lines 1, 2, 3, 4, 5, and 6. The corresponding solid points on the figure represent the non-inferior solutions in decision space, numbered 1 through 6. These solutions are summarized in Table (7). The coordinates x_1, x_2 for the respective non-inferior solutions are obtained from Figure (3).

Comment

Step 19 of 25

Summarize the non-inferior solutions generated by the constraint method as in Table (6).

Non-inferior solution	L_k	x_1	x_2	$Z_1 = 9x_1 + 6x_2$
1	−9	7.8	3.3	90
2	−6	6.8	4.5	88.2
3	−3	6	6	90
4	0	4.2	6	73.8
5	3	2.8	6	61.2
6	6	1.1	6	45.9

Comment

Step 20 of 25

Approximation of the non-inferior set having non-inferior solutions along the Z_2 axis by using the weighting method:

The grand objective function is formed by multiplying each objective function by a weighting factor and adding these weighted objective function terms together.
Write the grand objective function (Z^G).

$$\text{Maximize } Z^G = w_1 Z_1 + w_2 Z_2$$
$$= w_1(9x_1 + 6x_2) + w_2(3x_1 - 2x_2)$$

Comment

Step 21 of 25

For minimization objectives, the objective function is multiplied by −1 to change its sense to maximization. The relative weights on these objective functions are significant than their specific values.

$$\text{Maximize } Z^G = w_1(9x_1 + 6x_2) + w_2(3x_1 - 2x_2)$$

Construct a normal linear program by setting weights of 0.9 for w_1 and 0.1 for w_2 in the grand objective function.

$$\text{Maximize } Z^G = 0.9\times(9x_1 + 6x_2) + 0.1\times(3x_1 - 2x_2)$$
$$= 8.1x_1 + 5.4x_2 + 0.3x_1 - 0.2x_2$$
$$= 8.4x_1 + 5.2x_2$$

Comment

Step 22 of 25

Tabulate the grand objective function and their respective weights and the non-inferior solution that would result by optimizing the objective function as in Table (7).

Gradient	w_1	w_2	Objective function	Solution
1	1	0	$9x_1 + 6x_2$	D, E
2	0.9	0.1	$8.4x_1 + 5.2x_2$	E
3	0.8	0.2	$7.8x_1 + 4.4x_2$	E
4	0.7	0.3	$7.2x_1 + 3.6x_2$	E
5	0.6	0.4	$6.6x_1 + 2.8x_2$	E
6	0.5	0.5	$6x_1 + 2x_2$	E
7	0.4	0.6	$5.4x_1 + 1.2x_2$	E, F
8	0.3	0.7	$4.8x_1 + 0.4x_2$	F
9	0.2	0.8	$4.2x_1 - 0.4x_2$	F
10	0.1	0.9	$3.6x_1 - 1.2x_2$	A
11	0	1	$3x_1 - 2x_2$	A

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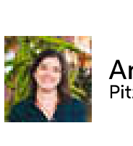

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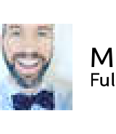

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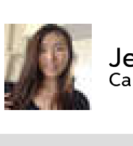

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Calculate the slope θ of the gradient line for the weight values of 0.9 for w_1 and 0.1 for w_2

$$\tan\theta = \frac{0.9}{0.1}$$
$$\theta = 83.65°$$

The line 3 makes an angle of 83.65 degree with the Z_1 axis.

Repeat the slope calculation with other weight values.

Line 1 is vertical and makes 90 degree angle with the Z_1 axis.

Line 2 is horizontal and makes 0 degree angle with the Z_1 axis.

Comment

Step 24 of 25

Plot the lines in the objective space. Gradient line 1 passes through D and E.

Gradient lines 3, 4, 5, and 6 pass through E.

Gradient line 7 passes through E and F.

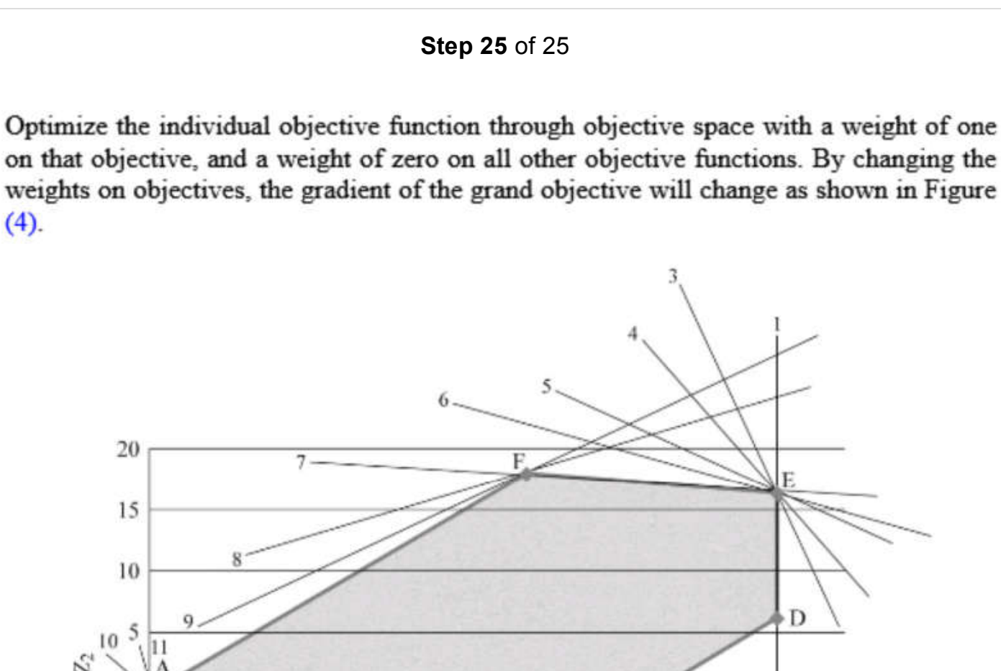
Gradient lines 8 and 9 pass through F.

Gradient lines 10 and 11 pass through A.

Comment

Step 25 of 25

Optimize the individual objective function through objective space with a weight of one on that objective, and a weight of zero on all other objective functions. By changing the weights on objectives, the gradient of the grand objective will change as shown in Figure (4).



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